



Cornell University®

Big Red Math Competition
Individual Round

Name: _____

Student ID: _____

10/25/2025

11:15am–01:00pm

INSTRUCTIONS — PLEASE READ THIS NOW

- **Write your Name and Student ID.**
- **Simplification.** Please simplify all answers as much as possible. For example, $\frac{\cos \pi}{\frac{1}{2}}$ can be simplified to -2 .
- This is a closed book exam. You are **NOT** allowed to use a calculator, computer, notes, or any other resources.
- All problems are worth the same in this round.
- **You have 105 minutes to complete this exam.**

Please sign below to indicate that you have read and agree to these instructions.

Signature, Print Name: _____

Answer Sheet:

- | | | | |
|----------|----------|-----------|-----------|
| 1. _____ | 5. _____ | 9. _____ | 13. _____ |
| 2. _____ | 6. _____ | 10. _____ | 14. _____ |
| 3. _____ | 7. _____ | 11. _____ | 15. _____ |
| 4. _____ | 8. _____ | 12. _____ | |
-

Please don't mark anything below on the page, it's for official use only.

Grader 1 Name:

Grader 2 Name:

Grader 3 Name:

Tally:

Tally:

Tally:

Score:

Score:

Score:

1. Alice and Bob are playing a best-of-seven series of basketball. Alice has won the first two games. If Alice and Bob are equally likely to win a game, what is the probability that Bob wins the series?
2. In convex quadrilateral $ABCD$, AC intersects BD at a right angle at point E . Also, $AB = CD$, $AD = CE$, and $DE = 1$. Find the length of BE .
3. An 11×11 square grid has 121 square cells. In how many ways can one select a 1×1 square A and a 2×2 square B from this grid such that A does not lie inside B , but A and B touch by an edge. (Touching by only a corner does not count.)
4. Steph Curry successfully made 20 of his 25 shots in his last game. During the game, the *most* consecutive shots he successfully made was k shots. What is the *minimum* value of k .
5. Using each one of 4 distinct digits a, b, c , and d exactly once, 24 numbers can be formed. What is the maximum possible number of multiples of 36 that can be among those 24 numbers?
6. For each real number r , let $\lfloor r \rfloor$ denote the largest integer that is less than or equal to r . Find the sum of all real numbers x such that $\lfloor 97 + 19x \rfloor = 97x + 19$.
7. Given two distinct points $P = (x_1, y_1)$ and $Q = (x_2, y_2)$ in the Cartesian plane, let D denote the distance between P and Q . Also, let T denote the *taxicab distance* between P and Q given by the formula $T = |x_1 - x_2| + |y_1 - y_2|$. What is the maximum possible value that the ratio $\frac{T}{D}$ can attain?
8. We call a subset S of $\{1, 2, \dots, 100\}$ *neat* if no integer in S divides another (distinct) integer in S . If A and B are disjoint *neat* subsets of $\{1, 2, \dots, 100\}$, what is the largest possible number of integers that are in either A or B .
9. Given a number n , we can write it as $b_0b_1\dots b_k$ in binary expansion ($b_0 \neq 0$). Define $f(n)$ as the number that has binary expansion $b'_0b'_1\dots b'_k$, where $b_i \neq b'_i$ for all i . This is, we interchange 0's by 1's in the binary expansion for n .
Determine how many positive integers less than 256 have the property that $f(n)$ is divisible by 3.
Note: For example, since 10 and 5 have binary expansions 1010 and 101 respectively, $f(10) = 5$ and $f(5) = 2$.
10. Let $p = 101$ be a prime number. For each integer $2 \leq k \leq p - 2$, let $1 \leq a_k \leq p - 1$ be the unique integer such that $(k^2 - k)a_k$ has remainder 1 when divided by p . Compute the remainder of $a_2 + a_3 + \dots + a_{p-2}$ when divided by p .
11. Let G be an undirected graph with 20 vertices and 100 edges. The number of ways to choose an unordered pair of disjoint edges is 4050. Find $\sum_{v \in G} \deg(v)^3$.
Note: Two edges are said to be *disjoint* if they don't share a common vertex. Also, for each vertex v , the value $\deg(v)$ denotes the number of edges that are connected to v .
12. Determine how many positive integers $n \leq 100$ satisfy that $(n + 1)!$ has exactly twice as many divisors as $n!$.
13. Points $X_1, \dots, X_{10}$ lie, in that order, on a line segment. Nine isosceles triangles are constructed, with bases $X_1X_2, X_2X_3, \dots, X_9X_{10}$, and with opposing angle equal to α . If all 9 opposing vertices happen to fall on the semicircumference of diameter X_1X_{10} , what is α ?

14. We say a positive integer n is *phenomenal* if there exists a polynomial P with integer coefficients such that $P(d) = \left(\frac{n}{d}\right)^2$ for all positive divisors d of n . Find the largest composite *phenomenal* number.
15. Let $n = 2025$. Let X denote the set of all possible integer sequences such that each element in the sequence is between 1 and n , inclusive. Given two sequences $x = (x_1, x_2, x_3, \dots)$ and $y = (y_1, y_2, y_3, \dots)$ in X , we say x and y are *friendly* if

$$\sum_{k=1}^{\infty} \frac{|x_k - y_k|}{n^k} \leq \frac{1}{n^n}.$$

Let S be a subset of X of minimum possible size such that every sequence in X is friendly with some sequence in S . If k denotes the number of unordered pairs of distinct sequences in S that are friendly with each other, compute the maximum possible value of k .